

Electromagnetic Corrections to $\pi\pi$ Scattering Lengths: Some Lessons for the Implementation of Meson Exchange Models

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Abstract

The leading (in chiral order) electromagnetic corrections to s-wave $\pi\pi$ scattering lengths are computed using chiral perturbation theory (ChPT). It is shown that contributions associated, not with one photon exchange, but rather with contact terms in the effective electromagnetic Lagrangian, dominate previously computed radiative corrections (which are higher order in the chiral expansion). These corrections bring experimental values into closer agreement with the results of ChPT to one-loop order. It is also pointed out that standard implementations of electromagnetism in the meson-exchange model framework omit such contact terms and that this omission, combined with experimental input, would lead to the erroneous conclusion that $\pi\pi \rightarrow \pi\pi$ exhibited very large strong isospin breaking. Implications for standard “electromagnetic subtraction” procedures, and for the construction of meson-

exchange models in general, are also discussed.

13.40.Ks,12.39.Fe,13.75.Lb,11.30.Rd

Over the past decade or so, there has been considerable debate between proponents of meson-exchange models and those models/methods involving explicit quark and gluon degrees of freedom, over which approach to treating strongly interacting few-body systems is the correct one. In the most general context, however, this debate is, in fact, meaningless. Indeed, one is free to restrict one's attention to a set of dynamical, low-lying hadronic degrees of freedom and write a low-energy effective Lagrangian, \mathcal{L}_{eff} , which involves explicitly only these degrees of freedom. So long as this Lagrangian is constructed in such a way as to be the *most general* such Lagrangian possessing all the exact symmetries of QCD and realizing the approximate chiral symmetries of QCD in the same way they are realized in QCD, it will, of course, be identical to QCD in its consequences for any processes involving only the explicitly considered degrees of freedom — provided, that is, the unknown coefficients accompanying the terms in \mathcal{L}_{eff} allowed by the symmetry arguments (called “low-energy constants”, or LEC's) are given the values they would have in QCD. One can then view typically implemented meson-exchange models as truncations of the most general such low-energy effective theory – truncations in which certain types of terms are omitted and in which the values of the LEC's accompanying other terms are not computed from QCD (usually an impossible task, at present), but rather are left free, to be determined phenomenologically. These truncations may, of course, do damage to the underlying theory. In the untruncated version, however, the effective theory fully incorporates QCD, despite having no explicit quark and gluon degrees of freedom. Their effect is present, but buried in the precise values of the LEC's which describe the non-renormalizable low-energy effective theory.

In order to set the context for the above remarks, it is useful to remember that there exists a general framework for constructing, for a given restricted set of hadronic degrees of freedom, the most general low-energy effective theory compatible with the symmetries and approximate symmetries of QCD [1,2]. A crucial feature of the resulting effective Lagrangians is the presence of contact interactions involving, in general, arbitrary numbers of hadronic fields. These contact interactions are present because, in constructing \mathcal{L}_{eff} , one has effectively integrated out high frequency components. Certain n-point Green functions gen-

erated by explicit quark and gluon loops in QCD then generate, from their high-momentum (short distance) components, effective interactions which appear point-like from the perspective of the effective theory. Such contact terms are almost always omitted in formulating meson-exchange models. However, to the extent that they correspond to terms allowed by the symmetries of QCD, they are a necessary consequence of QCD, and must be present in \mathcal{L}_{eff} , regardless of whether or not this complicates the phenomenological task of fixing the values of the full set of unknown LEC's relevant to a given process. Expanding the set of hadronic degrees of freedom included in \mathcal{L}_{eff} may alter the values of the LEC's (and introduce new LEC's, as well), but for any finite truncation of the hadronic degrees of freedom, such contact terms will necessarily be present. Moreover, as we will show in this letter using the example of electromagnetic contributions to $\pi\pi$ scattering, omission of these contact terms can lead to serious numerical errors in the treatment of the physics of a given problem.

Let us now illustrate the above comments by showing explicitly how the effects of electromagnetism (EM) in hadronic systems would be incorporated in the effective Lagrangian framework. The discussion will also serve to fix our notation. We adhere throughout to the general framework for constructing \mathcal{L}_{eff} developed in in Refs. [1–4] (see also Refs. [5,6] for excellent recent reviews).

If one imagines an effective theory involving, say, the pseudoscalar and vector mesons and octet baryons, then, as is well-known, one may make field choices for the particles such that, with B the standard octet baryon matrix, $S_{\mu\nu}$ and $V_{\mu\nu}$ the singlet and flavor-octet matrix for the vector mesons in the tensor field representation [7,8], $\pi \equiv \lambda^a \pi^a$ the standard octet pseudoscalar matrix, $U = \exp(i\pi/F)$, and $u = \exp(i\pi/2F)$ (with F a parameter which turns out to be the π decay constant in the chiral limit), the fields transform as

$$B \rightarrow h B h^\dagger \tag{1}$$

$$V_{\mu\nu} \rightarrow h V_{\mu\nu} h^\dagger \tag{2}$$

$$S_{\mu\nu} \rightarrow S_{\mu\nu} \tag{3}$$

$$U \rightarrow R U L^\dagger \tag{4}$$

under the chiral group $SU(3)_L \times SU(3)_R$ [1,2,9]. Here, h is defined (as a function of π^a and the left and right chiral transformation matrices, L and R) via

$$u \rightarrow R u h^\dagger = h u L^\dagger \quad (5)$$

and reduces to the ordinary $SU(3)_V$ transformation matrix $V = L = R$ when the chiral transformation lies in the vector subgroup $SU(3)_V$. In the chiral limit (no EM and $m_u = m_d = m_s = 0$), \mathcal{L}_{eff} consists of all terms involving the field variables and their derivatives which are invariant under the full chiral group (as well as under C, P, T and Lorentz transformations). The construction of \mathcal{L}_{eff} is, as usual, greatly simplified by introducing the covariant derivatives of the various fields, which, by construction, transform in the same way as the original fields, e.g.

$$\begin{aligned} D_\mu U &\equiv \partial_\mu U - i r_\mu U + i U \ell_\mu \rightarrow R(D_\mu U) L^\dagger \\ D_\mu B &\equiv \partial_\mu B + [\hat{V}_\mu, B] \rightarrow h(D_\mu B) h^\dagger \text{ etc.}, \end{aligned} \quad (6)$$

where $\hat{V}_\mu \equiv \frac{1}{2} [u^\dagger(\partial_\mu - i r_\mu)u + u(\partial_\mu - i \ell_\mu)u^\dagger]$ with \hat{V}_μ transforming as $\hat{V}_\mu \rightarrow h \hat{V}_\mu h^\dagger + h \partial_\mu h^\dagger$, where r_μ and ℓ_μ are the external right and left-handed sources which, for example, allow one to treat explicit couplings to photons and W bosons. (Since $r_\mu = v_\mu + a_\mu$, $\ell_\mu = v_\mu - a_\mu$, with v_μ and a_μ the external vector and axial vector spurces, the choice $v_\mu = -eQ A_\mu$, for example, with Q the quark charge matrix, generates the explicit couplings to photons.) The external field tensors, $L_{\mu\nu}$, $R_{\mu\nu}$, which transform as $L_{\mu\nu} \rightarrow L L_{\mu\nu} L^\dagger$, $R_{\mu\nu} \rightarrow R R_{\mu\nu} R^\dagger$, and their covariant derivatives can also occur in \mathcal{L}_{eff} . To facilitate construction of the couplings of the pseudoscalars to the baryons and vector mesons, it is also convenient to introduce, in addition to \hat{V}_μ above, the combination $\hat{A}_\mu \equiv \frac{i}{2} [u^\dagger(\partial_\mu - i r_\mu)u - u(\partial_\mu - i \ell_\mu)u^\dagger]$ which transforms as $\hat{A}_\mu \rightarrow h \hat{A}_\mu h^\dagger$. Similarly, the couplings of baryons and vector mesons to the external field tensors are simplified by employing

$$F_\pm^{\mu\nu} \equiv u L^{\mu\nu} u^\dagger \pm u^\dagger R^{\mu\nu} u \quad (7)$$

which transform as $F_{\pm}^{\mu\nu} \rightarrow h F_{\pm}^{\mu\nu} h^{\dagger}$. The basic ingredients for constructing \mathcal{L}_{eff} are then traces of products of the building blocks above and their covariant derivatives, where the terms are ordered inside a given trace in such a way as to produce manifest invariance.

EM and quark masses, which explicitly break the chiral invariance, are also easily incorporated by noting that the chiral symmetry breaking piece of \mathcal{L}_{QCD} , $\mathcal{L}_{\chi SB}$,

$$\mathcal{L}_{\chi SB} = -\bar{\psi}_L M_{LR} \psi_R - \bar{\psi}_R M_{RL} \psi_L + e A_{\mu} \left(\bar{\psi}_L Q_{LL} \gamma^{\mu} \psi_L + \bar{\psi}_R Q_{RR} \gamma^{\mu} \psi_R \right) \quad (8)$$

(where $M_{LR} = M_{RL} = M$ is the quark mass matrix, and $Q_{LL} = Q_{RR} = Q$ is the quark charge matrix) would be invariant if M_{LR} *etc.* were thought of as spurions transforming as indicated by the subscripts, *i.e.*, $M_{LR} \rightarrow L M_{LR} R^{\dagger}$, *etc.*. The chiral symmetry breaking terms in \mathcal{L}_{eff} are then generated by including explicit factors of M , Q to produce terms which are invariant under the action of the joint field and spurion transformation rules above. (For convenience of use in the baryon and vector meson sectors, one may use, in place of M , Q , M_{\pm} and Q_{\pm} , where

$$\begin{aligned} M_{\pm} &= \frac{1}{2} \left(u^{\dagger} M_{RL} u^{\dagger} \pm u M_{LR} u \right) \\ Q_{\pm} &= \frac{1}{2} \left(u^{\dagger} Q_{RR} u \pm u Q_{LL} u^{\dagger} \right) \end{aligned} \quad (9)$$

which transform as $M_{\pm} \rightarrow h M_{\pm} h^{\dagger}$, $Q_{\pm} \rightarrow h Q_{\pm} h^{\dagger}$.)

Let us now employ the above machinery to incorporate EM in \mathcal{L}_{eff} . The first class of contributions is familiar from the conventional treatments of EM in the meson-exchange framework. It consists of those graphs involving explicit photons, either via one-photon exchange ($1\gamma E$) or through radiative corrections (*i.e.* graphs with photon loops). The photon couplings of the mesons and baryons are generated by the presence of the external vector field v_{μ} , which includes the photon, in the covariant derivatives of the baryon and meson fields, and also by the possibility of couplings involving the external field tensors, $L^{\mu\nu}$, $R^{\mu\nu}$ (or, equivalently, $F_{\pm}^{\mu\nu}$). An example of the latter would be [8]

$$\mathcal{L}_{\gamma V} = \frac{F_V}{2\sqrt{2}} \text{Tr} [V_{\mu\nu} F_+^{\mu\nu}] \quad (10)$$

a P,C,T, Lorentz and chiral invariant term which, at zeroth order in the pseudoscalar fields, couples the ρ^0 and ω_8 to the photon. The LEC F_V is then the vector meson decay constant in the chiral limit. As usual, its value is not fixed by the symmetry arguments used in obtaining $\mathcal{L}_{\gamma V}$, which arguments tell us only that a term of the form $\mathcal{L}_{\gamma V}$ must be present in \mathcal{L}_{eff} . The value of F_V is to be fixed from experiment, or calculated explicitly in QCD. Note that $\mathcal{L}_{\gamma V}$, upon expanding the exponentials in u, u^\dagger appearing in $F_+^{\mu\nu}$, also involves contact couplings of a photon to a single vector meson plus arbitrary numbers of pseudoscalars.

The explicit photon-exchange and radiative correction contributions to a given process do not, however, exhaust the set of EM effects. As usual, because we have an effective low-energy theory, certain high frequency photon-exchange and -loop contributions are effectively frozen out and represented by EM contact terms in \mathcal{L}_{eff} , *i.e.* terms which involve two powers of the quark charge matrix, but which involve no explicit photons. It is easy to see, by construction, that such terms must appear in \mathcal{L}_{eff} . If we restrict our attention to terms which are momentum-independent then, for example, for the pseudoscalar sector, there is a unique such term,

$$\mathcal{L}_{\pi,EM}^{(0)} = c_\pi F^2 \text{Tr} [U_{RL} Q_{LL} U_{LR}^\dagger Q_{RR}] \quad (11)$$

where the insertion of the factor F^2 is conventional, the superscript indicates the chiral order (no derivatives and no powers of M , in this case), and the chiral transformation labels have been included as subscripts so one can see the manifest chiral invariance of $\mathcal{L}_{\pi,EM}^{(0)}$. This term generates contributions to the π^\pm, K^\pm masses and reproduces (in a very efficient manner) the current algebra relations for the pseudoscalar EM self-energies [10].

Similarly, if one wished to consider EM contributions to ρ - ω mixing, one would have not only the usually considered contributions, associated with an intermediate one-photon state, but also contributions from terms in \mathcal{L}_{eff} of the form

$$\begin{aligned} \mathcal{L}_{V,EM}^{(0)} = & c_V^{(1)} \text{Tr} [V_{\mu\nu} V^{\mu\nu} Q_+^2] + c_V^{(2)} \text{Tr} [V_{\mu\nu} Q_+^2] S^{\mu\nu} + c_V^{(3)} \text{Tr} [V_{\mu\nu} Q_+ V^{\mu\nu} Q_+] + \\ & c_V^{(4)} \text{Tr} [V_{\mu\nu} Q_+] \text{Tr} [V^{\mu\nu} Q_+] + \dots \end{aligned} \quad (12)$$

where $+\dots$ refers to other zeroth order EM terms which do not contribute to ρ - ω mixing and which, hence, have not been written down explicitly.

EM zeroth order contact terms also exist for the baryons [11,12]:

$$\begin{aligned}\mathcal{L}_{B,EM}^{(0)} = & c_B^{(1)}\text{Tr}[\bar{B}Q_+^2B] + c_B^{(2)}\text{Tr}[\bar{B}BQ_+^2] + c_B^{(3)}\text{Tr}[Q_+\bar{B}Q_+B] \\ & + c_B^{(4)}\text{Tr}[\bar{B}Q_+] \text{Tr}[BQ_+] + c_B^{(5)}\text{Tr}[Q_+^2] \text{Tr}[\bar{B}B]\end{aligned}\quad (13)$$

(not all of these terms are linearly independent [12], but we show the full set of manifestly chiral invariant terms that can be constructed, for completeness).

Van Kolck, Friar and Goldman have, similarly, demonstrated the presence of such EM contact terms for πN interactions [13].

All of the above EM terms are *necessarily* present in \mathcal{L}_{eff} . Any attempt to evaluate EM effects ignoring them will result in incorrect results, *i.e.*, results incompatible with the constraints of QCD.

Let us now turn to the case of EM contributions to $\pi\pi$ scattering in order to demonstrate that the contact (“invisible photon”) contributions can, in fact, be numerically crucial. We will make non-trivial use of the chiral counting, or low-energy expansion, in which contributions to physical observables are organized in a joint series in momenta and quark masses. Because of the leading chiral result that $m_\pi^2 \equiv q_\pi^2$ is proportional to $(m_u + m_d)$, it is necessary to count M as $\mathcal{O}(q^2)$. The terminology “zeroth order” then means no momenta and no quark masses, “second order” two powers of momenta or one power of quark masses, etc.. Using this counting, and the fact that the charged meson-photon couplings are $\mathcal{O}(q)$ and the photon propagator $\mathcal{O}(q^{-2})$, the $1\gamma E$ graphs in $\pi\pi$ scattering are seen to be $\mathcal{O}(q^0)$. The leading strong contributions to $\pi\pi$ scattering are produced by the lowest order part of the effective strong Lagrangian

$$\mathcal{L}_{st}^{(2)} = \frac{F^2}{4}\text{Tr}[D_\mu U D^\mu U^\dagger] + B_0 \frac{F^2}{2}\text{Tr}[MU^\dagger + UM]. \quad (14)$$

The LEC, F , is the π decay constant in the chiral limit, while the second LEC, B_0 , determines the quark condensate in the chiral limit via $\langle 0|\bar{q}q|0\rangle = -B_0 F^2$. $\mathcal{L}_{st}^{(2)}$ gives rise, at leading

order, to the Weinberg results for the $\pi\pi$ scattering lengths and slope parameters [14]. Using the Weinberg counting argument [3], one easily sees that the radiative corrections to the tree-level strong scattering graphs are $\mathcal{O}(q^2)$ in the chiral counting.

It should be noted that, despite the arguments above, which show the necessity of EM contributions not involving explicit photons in any low-energy effective hadronic theory, standard implementations in the meson-exchange-model framework typically include only contributions associated with the presence of explicit photons. Frequently, moreover, it is only the $1\gamma E$ contributions which are considered, as, for example, in the “EM subtraction” performed on NN scattering data to determine the strong charge-independence- and charge-symmetry-breaking scattering length differences $a_{pp} - a_{np}$ and $a_{pp} - a_{nn}$. Certain other calculations do include EM radiative corrections, as in the case of Morrison’s treatment of EM corrections to πNN couplings [15] or, particularly pertinent to the case at hand, Roig and Swift’s treatment of EM corrections to the $\pi\pi$ s-wave scattering lengths [16] (in which $1\gamma E$ contributions, including the effect of the π EM form factor, as well as the full set of radiative and bremsstrahlung corrections, were taken into account). However, apart from corrections to $\pi\pi$ scattering [16] and pionium decay [17] associated with the use of different masses for the neutral and charged pions (since this splitting is known to be essentially pure EM [18], this amounts to taking into account the effect of those parts of the EM contact interaction of Eqn. (11) second order in the π fields), corrections associated with the EM contact interactions have not been taken into account. In light of the discussions above, we see that such treatments of EM must, in fact, be inconsistent.

It is particularly easy to expose the problems of such treatments in the case of $\pi\pi$ scattering. To see this, note that $\mathcal{L}_{\pi,EM}^{(0)}$ contains π^4 vertices and hence produces contributions to $\pi\pi$ scattering amplitudes zeroth order in the chiral expansion. As noted above, the radiative corrections are, in contrast, second order, and hence expected to be considerably smaller (as will be borne out by the results below). Moreover, the set of radiative corrections do not even exhaust the $\mathcal{O}(q^2)$ EM contributions to $\pi\pi$ scattering; there exist one-loop graphs involving

vertices from $\mathcal{L}_{\pi,EM}^{(0)}$ and tree graphs involving vertices from $\mathcal{L}_{\pi,EM}^{(2)}$ (where the superscript is again the chiral order; for the form of $\mathcal{L}_{\pi,EM}^{(2)}$ see Ref. [19]), all of which contain no explicit photon but nonetheless produce EM contributions at $\mathcal{O}(q^2)$. It is thus first, inconsistent to include only the $\mathcal{O}(q^0)$ photon exchange contributions without including the $\mathcal{O}(q^0)$ contributions from $\mathcal{L}_{\pi,EM}^{(0)}$ and, second, inconsistent to include the radiative corrections without also including the other contributions, without explicit photons, of the same chiral order.

In this letter, we evaluate the $\mathcal{O}(q^0)$ $\mathcal{L}_{\pi,EM}^{(0)}$ contributions to the s-wave $\pi\pi$ scattering lengths, and point out the problems that would ensue from an incomplete “EM subtraction”, *i.e.* one involving only the $1\gamma E$ contributions. We will return to the full set of $\mathcal{O}(q^2)$ EM contributions in a later publication [20]. Note that *no* version of the typical meson-exchange-model treatment involving only graphs with explicit photons would incorporate the effects of the $\mathcal{O}(q^0)$ EM contact terms. It is usually said that the form factors employed in such models are a phenomenological means of incorporating short-distance effects. However, such form factors, in the low-energy effective theory, are generated by higher-chiral-order corrections (loops, as well as tree graphs involving higher order vertices) to the leading tree-level vertices. Hence, in the case of the EM interactions, incorporating such form factors in graphs with explicit photons produces only higher order corrections, which begin at $\mathcal{O}(q^2)$ and hence could not possibly account for the contributions associated with the $\mathcal{O}(q^0)$ contact terms.

We now turn to the evaluation of the non- $1\gamma E$ $\mathcal{O}(q^0)$ EM contributions to $\pi\pi$ scattering. In order to optimize our numerical accuracy, we will work with the $SU(2) \times SU(2)$ effective Lagrangian, involving the pions alone, the $SU(2) \times SU(2)$ chiral symmetry being much better respected than $SU(3) \times SU(3)$. The form of $\mathcal{L}_{\pi,EM}^{(0)}$ is the same as in Eqn. (11) except that U is now given by $U = \exp(i\vec{\tau} \cdot \vec{\pi}/F)$, Q is the $u - d$ quark charge matrix, and we have three, rather than eight, pseudoscalar fields. The part of $\mathcal{L}_{\pi,EM}^{(0)}$ second order in the fields is then easily shown to be $-2c_\pi \pi^+ \pi^-$ which produces the well-known non-zero EM contribution to the charged pion mass, $(\delta m_{\pi^\pm}^2)_{EM} = 2c_\pi$. Since, as is also well-known [18], the $\pi^\pm - \pi^0$ mass splitting is essentially purely EM, this fixes the value of c_π : $c_\pi = (m_{\pi^+}^2 - m_{\pi^0}^2)/2$. Expanding,

similarly, to fourth order in the fields, one finds a contribution

$$\frac{c_\pi}{3F^2} \left[2\pi^{02}\pi^+\pi^- + 4\pi^{+2}\pi^{-2} \right] . \quad (15)$$

With the definitions

$$T(s, t) = 32\pi \sum_{\ell=0}^{\infty} (2\ell+1) P_\ell(\cos(\theta_{CM})) t_\ell(s) \quad (16)$$

for the partial wave amplitudes, and

$$\text{Re}(t_0(s)) = a_0 + \mathcal{O}(q_{CM}^2) \quad (17)$$

for the s-wave scattering lengths, where θ_{CM} , q_{CM} are the scattering angle and magnitude of the three-momentum in the CM frame, one obtains the following $1\gamma E$ -subtracted s-wave scattering lengths at leading order in the EM chiral expansion

$$\begin{aligned} a_0(00; 00) &= \frac{1}{3}a_0^0 + \frac{2}{3}a_0^2 \\ a_0(+0; +0) &= \frac{1}{2}a_0^2 + \frac{c_\pi}{24\pi F^2} \\ a_0(+--; 00) &= \frac{1}{3}a_0^0 - \frac{1}{3}a_0^2 + \frac{c_\pi}{24\pi F^2} \\ a_0(+--; +-) &= \frac{1}{3}a_0^0 + \frac{1}{6}a_0^2 + \frac{c_\pi}{6\pi F^2} \\ a_0(++; ++) &= a_0^2 + \frac{c_\pi}{6\pi F^2} \end{aligned} \quad (18)$$

where $a_0^{0,2}$ are the strong $I = 0, 2$ s-wave scattering lengths and the arguments of $a_0^{0,2}$ refer to the initial and final charge states (e.g., $(00; 00)$ means the process $\pi^0\pi^0 \rightarrow \pi^0\pi^0$). At lowest chiral order, $a_0^0 = 7m_\pi^2/32\pi F^2 = 0.16$ and $a_0^2 = -m_\pi^2/16\pi F^2 = -0.045$, the Weinberg values for the s-wave $I = 0, 2$ scattering lengths. The $\mathcal{O}(q^4)$ strong corrections to these relations were worked out long ago by Gasser and Leutwyler [7], producing the usually quoted one-loop values

$$\begin{aligned} (a_0^0)_{1-loop} &= 0.20 \pm 0.01 \\ (a_0^2)_{1-loop} &= -0.042 \pm 0.008 . \end{aligned} \quad (19)$$

Using these values, one finds that the EM contact corrections range in size from 0 (for $\pi^0\pi^0 \rightarrow \pi^0\pi^0$) to 10% (for $\pi^+\pi^+ \rightarrow \pi^+\pi^+$). These corrections, of course, also break the familiar isospin relations between the various amplitudes. We discuss the consequences of this fact below. Note that the $1\gamma E$ -subtracted radiative corrections to the scattering lengths in Eqn. (18) are [16] 0%, -0.2% , -0.6% , -1.3% and $+2.2\%$, respectively. As expected from the chiral counting, these are much smaller than the $\mathcal{O}(q^0)$ contact corrections. We stress again that one should not add the radiative corrections of Ref. [16] to those of Eqn. (18) since there are additional $\mathcal{O}(q^2)$ EM corrections that have yet to be accounted for.

Let us now illustrate the dangers of ignoring the presence of the contact interactions in \mathcal{L}_{eff} . We imagine (though this is unlikely to be the case in practice any time in the near future) having available experimental data on all of the processes listed above, and having performed a standard “EM subtraction”, *i.e.* having removed the $1\gamma E$ contributions to the amplitude. (Recall that, because of the Coulomb pole, one must perform the $1\gamma E$ extraction in order to even be able to define the scattering lengths in the first place.) If we (erroneously) assumed that this had removed all EM effects, we could attempt to determine the isospin breaking in the strong interaction contributions by determining the discrepancies in the values of a_0^0 , a_0^2 extracted from various combinations of the $1\gamma E$ -subtracted amplitudes which would be equal in the isospin limit. (The analogue to the usual procedures used to extract strong interaction charge-independence- and charge-symmetry-breaking NN observables should be obvious.) For example, in the isospin limit,

$$a_0^2 = 2 [a_0(00; 00) - a_0(+--; +)] = 2 [a_0(+--; +) - a_0(+--; 00)] \quad . \quad (20)$$

Using Eqns. (18),(19), however, one would find

$$\begin{aligned} 2 [a_0(00; 00) - a_0(+--; +)] &= -0.0507 \\ 2 [a_0(+--; +) - a_0(+--; 00)] &= -0.0355 \end{aligned} \quad (21)$$

a discrepancy of 43%. Similarly,

$$3 \left[a_0(00; 00) - \frac{2}{3} a_0(++; ++) \right] = 0.191$$

$$3 \left[a_0(+--;+-) - \frac{1}{6} a_0(++;++) \right] = 0.211 \quad (22)$$

(where both expressions would be equal to a_0^0 in the isospin limit), an 11% discrepancy. One would then conclude that there was very large strong (*i.e.* generated by $m_d - m_u \neq 0$) isospin breaking in $\pi\pi \rightarrow \pi\pi$, a conclusion which, in fact, is completely erroneous, as noted already above [7].

It should be pointed out that the discrepancies displayed in Eqns. (21), (22) are the largest possible, *i.e.* that all other “extractions” of a_0^0 , a_0^2 would lie in the ranges bracketed by the quoted values. If we consider the processes actually used in the latest experimental extractions ($\pi^+\pi^+ \rightarrow \pi^+\pi^+$ and $\pi^+\pi^- \rightarrow \pi^0\pi^0$, obtained via an analysis of $\pi^+p \rightarrow \pi^+\pi^+n$ and $\pi^-p \rightarrow \pi^0\pi^0n$ near threshold [21]), then the corrections are smaller. Note that the $1\gamma E$ contributions are very forward peaked and would not affect the experimental analysis, whereas the contact interactions are purely s-wave, and hence would. If we subtract the contact EM contaminations of $\mathcal{O}(q^0)$, the corrections to a_0^0 , a_0^2 are -0.0076 and -0.0043 , respectively, lowering the central value of a_0^0 extracted experimentally from 0.21 to 0.20 and that of a_0^2 from -0.031 to -0.035 (4% and 14% corrections, respectively), in both cases, in improved agreement with the predictions of (strong) ChPT to one loop. Although these corrections are smaller than the existing ± 0.07 , ± 0.007 errors on the experimental extractions, this is only marginally so for the a_0^2 case, which is the more favorable of the two for future improvements in the accuracy of the experimental determination, as explained in Ref. [21]. It should also be noted that there will be additional isospin-breaking effects in $\pi N \rightarrow \pi\pi N$ which have not yet been considered in the analysis of Ref. [21], and which could further alter the extracted values. Nonetheless, one should bear in mind that, without subtracting the known $\mathcal{O}(q^0)$ EM contact contributions, the experimental values of a_0^0 and a_0^2 extracted from $\pi^+\pi^+ \rightarrow \pi^+\pi^+$ and $\pi^+\pi^- \rightarrow \pi^0\pi^0$ would be expected to be, not 0.20 and -0.042 , but 0.21 and -0.038 , respectively.

In conclusion, we have considered the $\mathcal{O}(q^0)$ EM, non- $1\gamma E$ contributions to the $\pi\pi$ s-wave scattering lengths. The corrections are, in some cases, quite large (a possibility noted

previously by Gasser [22] and associated with the smallness of the strong scattering lengths, which vanish in the chiral limit). We find that, accounting for these corrections, the extracted values of a_0^0 and a_0^2 are brought into closer agreement with the results of ChPT to 1-loop. Moreover, the calculation demonstrates the unavoidable importance, in making EM subtractions, of considering not only the $1\gamma E$ and radiative corrections, but also those contributions associated with EM contact terms (involving no explicit photons), which contact terms are **necessarily** present in any low-energy effective hadronic theory representing the physics of the standard model. The implications for evaluating EM corrections to other strong interaction observables (such as the πNN couplings and NN scattering lengths) are obvious.

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REFERENCES

- [1] S. Coleman, J. Wess and B. Zumino, Phys. Rev. **177** (1969) 2239
- [2] C.G. Callen, S. Coleman, J. Wess and B. Zumino, Phys. Rev. **177** (1969) 2247
- [3] S. Weinberg, Physica **96A** (1979) 327
- [4] J. Gasser and H. Leutwyler, Nucl. Phys. **250** (1985) 465, 517, 539
- [5] G. Ecker, Prog. Part. Nucl. Phys. **35** (1995) 1
- [6] V. Bernard, N. Kaiser and U.-G. Meissner, Int. J. Mod. Phys. **E4** (1995) 193
- [7] J. Gasser and H. Leutwyler, Ann. Phys. **158** (1984) 142
- [8] G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. **B321** (1989) 311
- [9] H. Georgi, “Weak Interactions and Modern Particle Theory” (Benjamin, Menlo Park, 1984)
- [10] R. Dashen, Phys. Rev. **183** (1969) 1245
- [11] S. Coleman and S. Glashow, Phys. Rev. Lett. **6** (1961) 423
- [12] R.F. Lebed and M.A. Luty, Phys. Lett. **B329** (1994) 479
- [13] U. van Kolck, J. Friar and T. Goldman, Phys. Lett. **B371** (1996) 169
- [14] S. Weinberg, Phys. Rev. Lett. **17** (1966) 616
- [15] L.K. Morrison, Ann. Phys. **50** (1968) 6
- [16] F.S. Roig and A.R. Swift, Nucl. Phys. **B104** (1976) 533
- [17] U. Moor, G. Rasche and W.S. Woolcock, Nucl. Phys. **A587** (1995) 747
- [18] J. Gasser and H. Leutwyler, Phys. Rep. **C87** (1982) 77
- [19] R. Urech, Nucl. Phys. **B433** (1995) 234

- [20] K. Maltman and C. Wolfe, in progress
- [21] V. Bernard, N. Kaiser and U.-G. Meissner, Nucl. Phys. **B457** (1995) 147
- [22] J. Gasser, “The $\pi\pi$ Scattering Amplitude in Chiral Perturbation Theory”, in “The Second *DAΦNE* Physics Handbook”, INFN, Frascati, eds. L. Maiani, G. Pancheri and N. Paver